Particle Production in the Early Universe

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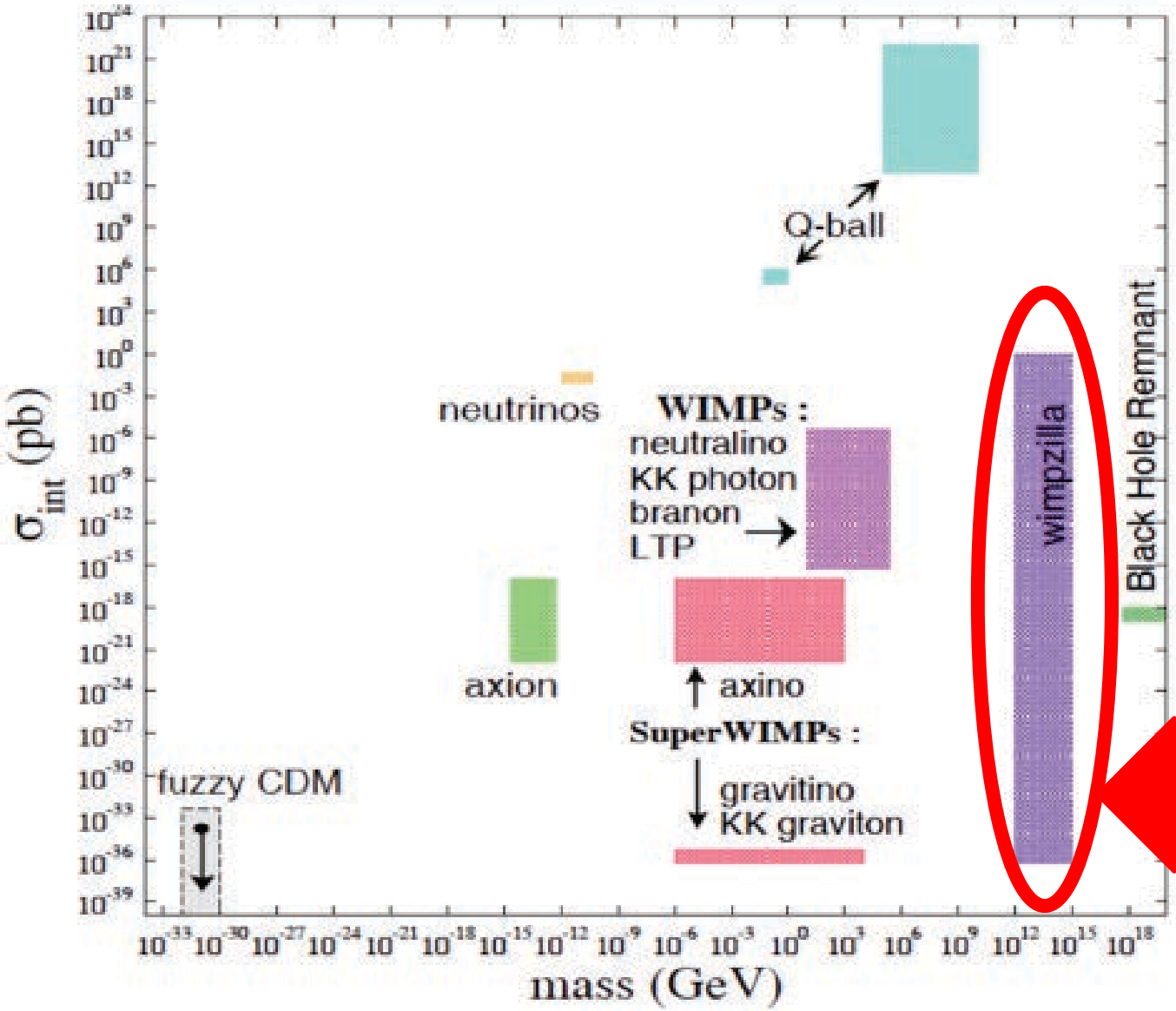
Dark Matter Candidates



This is where we are looking at.

Wimpzilla=Wimp+Godzilla

WIMPZILLAS!

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Figure 7. Dark matter may be much more massive than usually assumed, much more massive than wimpy WIMPS, perhaps in the WIMPZILLA class.

Abstract. There are many reasons to believe the present mass density of the universe is dominated by a weakly interacting massive particle (WIMP), a fossil relic of the early universe. Theoretical ideas and experimental efforts have focused mostly on production and detection of *thermal* relics, with mass typically in the range a few GeV to a hundred GeV. Here, I will review scenarios for production of *nonthermal* dark matter. Since the masses of the nonthermal WIMPS are in the range 10^{12} to 10^{16} GeV, much larger than the mass of thermal wimpy WIMPS, they may be referred to as WIMPZILLAS. In searches for dark matter it may be well to remember that "size does matter."

Wimpzilla

- Two necessary conditions
- 1. the WIMPZILLA must be stable, or at least have a lifetime much greater than the age of the universe.
- 2. it must not have been in equilibrium when it froze out (it is not a thermal relic), otherwise $\Omega_X h^2$ would be much larger than one.

Superheavy Dark Matter

- Weight: 10^{12} to 10^{16} GeV
- ~Hubble scale during inflation
- Related Models: Planckian Interacting Dark Matter, SUPERWIMP, FIMP, FIMPZILLA
- Origin: Supersymmetry breaking theories, Kaluza Klein theory of extra dimensions, string inspired models
- Production Mechanism: time dependent background

Production Time

- During inflation
- At inflation to matter dominated/radiation dominated era

Production Rate

- $m_X \ll H \;\; n_X \sim H^3$ **Ford 1987**

  $m_X \gg H \;\; n_X \sim \exp(-\# m_X/H)$ **Chung 2003**

  $n_X \sim H^3$ **Ema, Nakayama, Tang 2018**

Superheavy Dark Matter

- Action

$$S_X = -\int d^4x \sqrt{-g}\left[\frac{1}{2}\left(\partial_\mu X\right)^2 + \frac{1}{2}m_X^2 X^2 + \frac{1}{2}\xi R X^2\right]$$

- Equation of Motion

$$\ddot{X} + 3H\dot{X} - \frac{1}{a^2}\nabla^2 X + M_X^2 X = 0$$

$$M_X \equiv m_X^2 + \xi R$$

Superheavy Dark Matter

- Quantization

$$X = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} a^{-3/2} [f_k a_{\mathbf{k}} + f_k^* a^\dagger_{-\mathbf{k}}]$$

- The mode function satisfies

$$\ddot{f}_k + w_k^2(t) f_k = 0$$

$$w_k(t) = \sqrt{\frac{k^2}{a^2} + m^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}}$$

Standard Treatment

- Common Choice of Basis to Study Particle Production

$$f_k(t) = \alpha_k(t)\tilde{f}_k(t) + \beta_k(t)\tilde{f}_k^*(t)$$

- Basis $$\tilde{f}_k(t) \equiv \frac{1}{\sqrt{2w_k(t)}} e^{-i\int^t w_k}$$

- Coupled Differential Equation

$$\begin{pmatrix} \dot{\alpha}_k(t) \\ \dot{\beta}_k(t) \end{pmatrix} = \frac{\dot{\omega}_k(t)}{2\omega_k(t)} \begin{pmatrix} 0 & e^{2i\int^t \omega_k} \\ e^{-2i\int^t \omega_k} & 0 \end{pmatrix} \begin{pmatrix} \alpha_k(t) \\ \beta_k(t) \end{pmatrix}$$

Standard Treatment

- When $\alpha_k \simeq 1 \quad \beta_k \simeq 0$
- The previous coupled differential equation can be integrated

$$\beta_k \simeq \int d\tau \frac{w_k'}{2w_k} \exp\left(-2i \int^{\tau} w_k(\tau') d\tau' \right)$$

- In general very difficult to integrate
- A lot of approximations should be used

Stokes Line Method

- dS has a well defined particle number both at the beginning and the end, but not in the middle.
- We want to have a natural basis in which there is not much oscillations in the particle number in the time dependent background.
- Superadiabatic Basis

1405.0302 R. Dabrowski, G. V. Dunne

Stokes Line Method

- Dingle discovered a remarkable universal large-order behaviour for the adiabatic expansion.
- Define the "singulant" variable

$$F_k(t) = 2i \int_{t_c}^{t} w_k(t')dt' \qquad w_k(t_c) = 0$$

- Then the Basis $$W_k(t) = w_k(t) \sum_{l=0}^{\infty} \varphi_k^{(2l)}(t)$$
- Universal Large Order Behaviour

$$\varphi_k^{(2l+2)} \sim -\frac{(2l+1)!}{\pi F_k^{2l+2}}, \quad l \gg 1$$

Stokes Line Method: Summary

- Phase $$\Theta_k(t) = -i \int_{t_c}^{t} w_k(t')dt'$$
- Dingle's singulant variable $$F_k(t) = 2\Theta_k(t)$$
- Solution

$$f_k(t) \approx \exp[-\Theta_k(t_i)] \frac{\exp[\Theta_k(t)] + iS_k(t)\exp[-\Theta_k(t)]}{\sqrt{2\omega_k}}$$
$$= \frac{1}{\sqrt{2\omega_k}} \left\{ \exp\left(-i \int_{t_i}^{t} \omega_k dt' \right) - iS_k(t)\exp[-F_k(t_i)]\exp\left(i \int_{t_i}^{t} \omega_k dt' \right) \right\}$$

- Stokes multiplier function

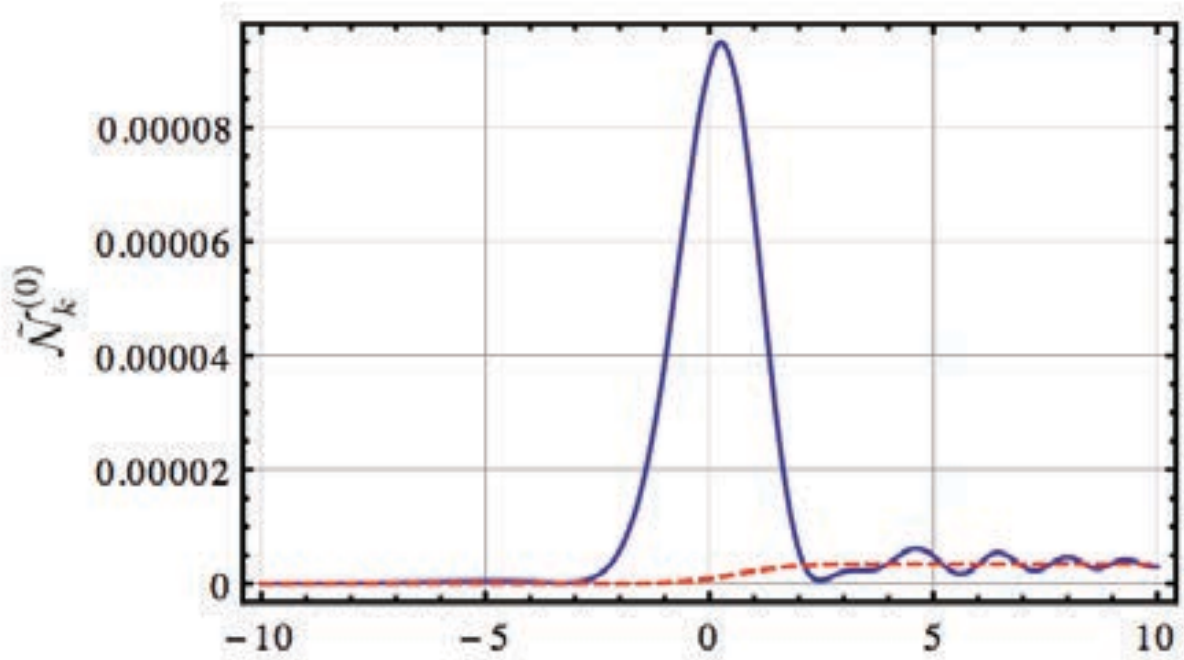
$$S_k(t) = \frac{1}{2}\left[1 + \mathrm{Erf}\left(\frac{-\mathrm{Im}F_k(t)}{\sqrt{2|\mathrm{Re}F_k(t)|}} \right)\right]$$

Stokes Line Method: Summary

- The moment when $\mathrm{Im}\, F_k(\mathrm{t}) = 0$ corresponds to the emergence of the negative-frequency part of the mode function, and the set of complex t satisfying this condition forms the Stokes line

Example

- Blue Line: Inappropriate Choice of Basis
- Red Dashed Line: Superadiabatic Basis



Particle Production of Inflation-Minkowski

- Scale Factor

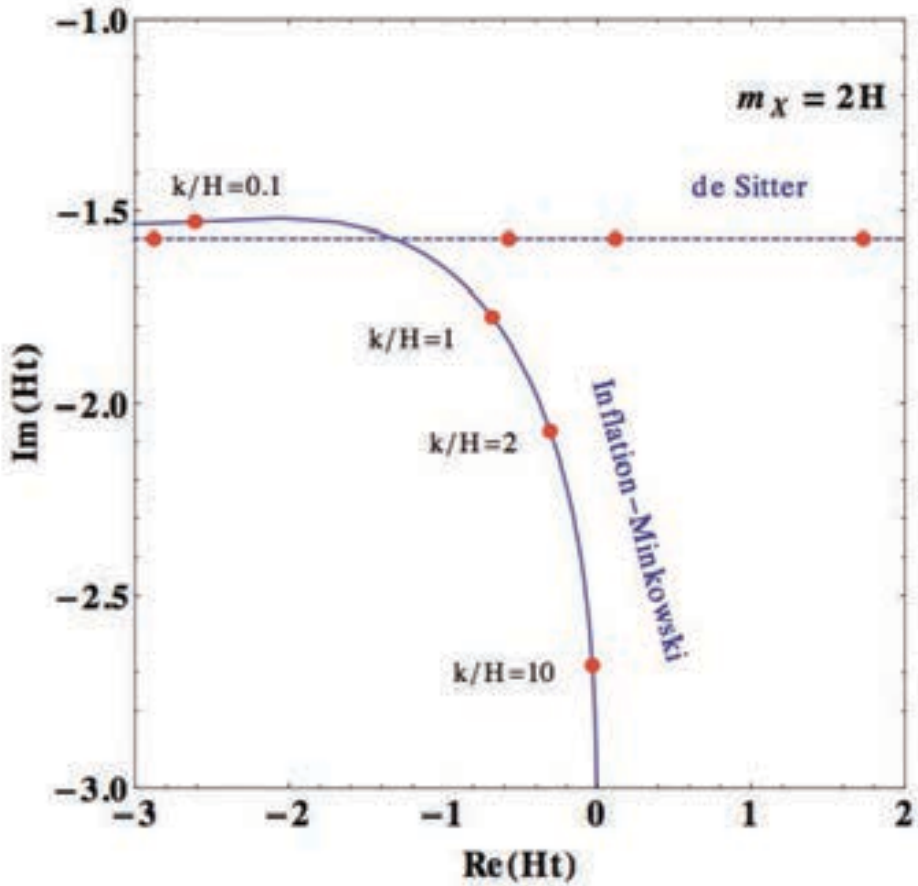
$$a(t) = \frac{e^{Ht}}{1 + e^{Ht}}$$

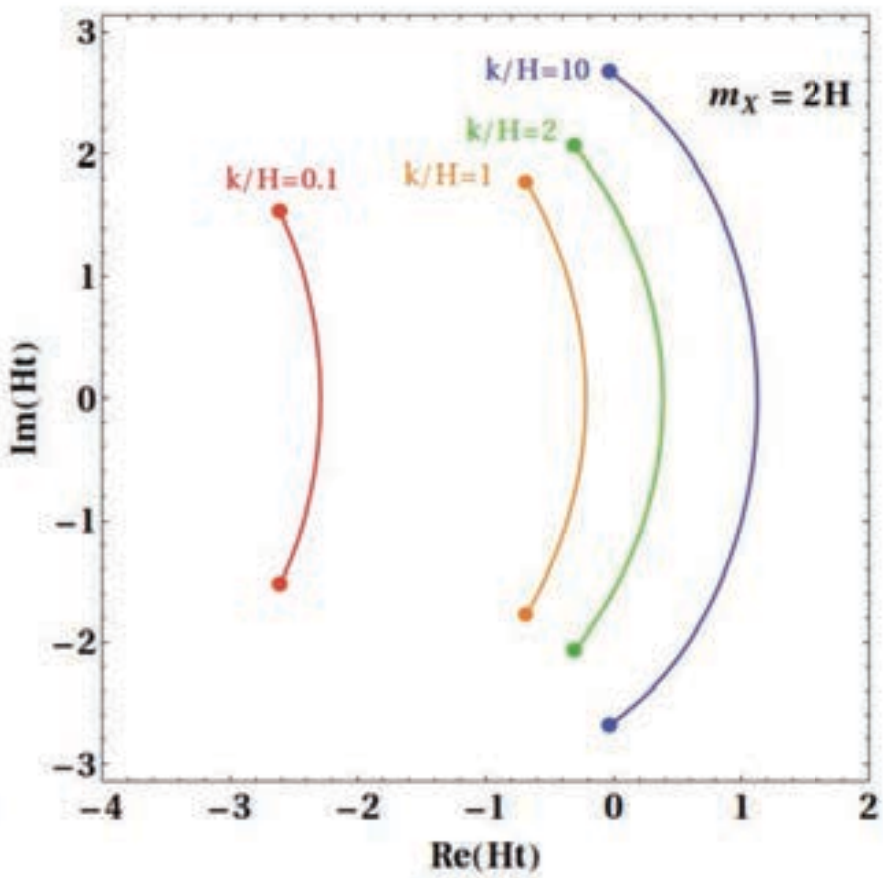
- Stokes Line Method

$$f_k(t) = \frac{\exp(-i\int_{t_i}^{t}\omega_k dt') - iS_k(t)\exp(i\int_{t_c}^{t_c^*}\omega_k dt')\exp(i\int_{t_i}^{t}\omega_k dt' + i\phi)}{\sqrt{2\omega_k}}$$

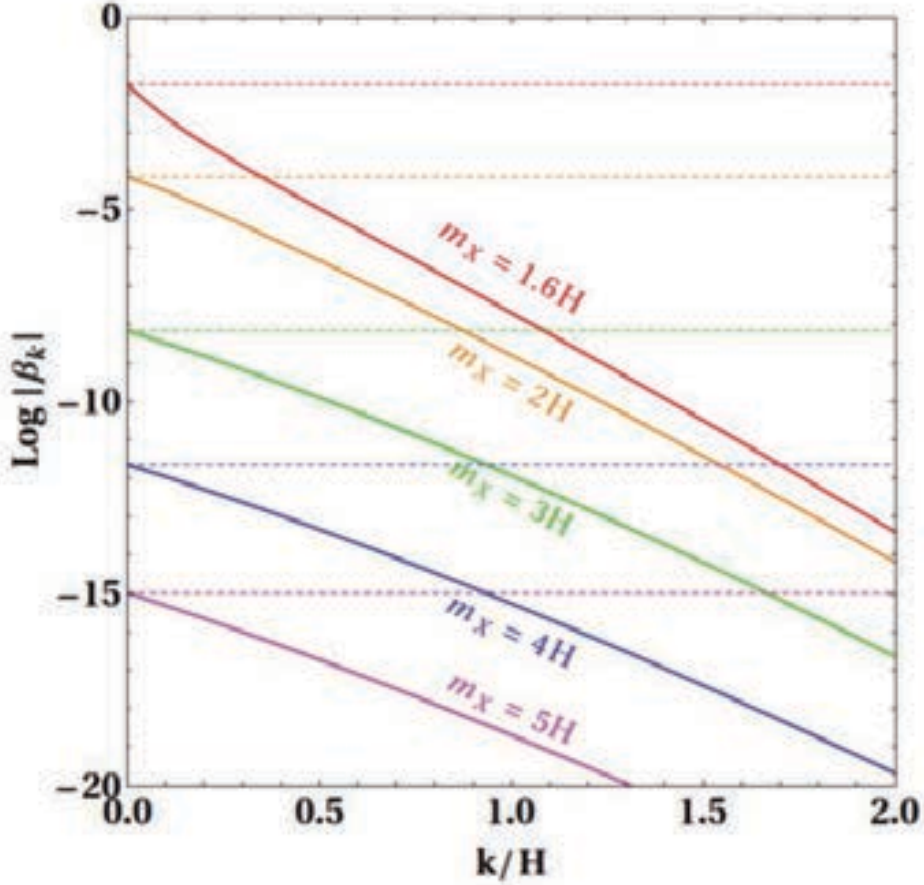
- t_c: the complex time satisfying $w_k(t_c) = 0$

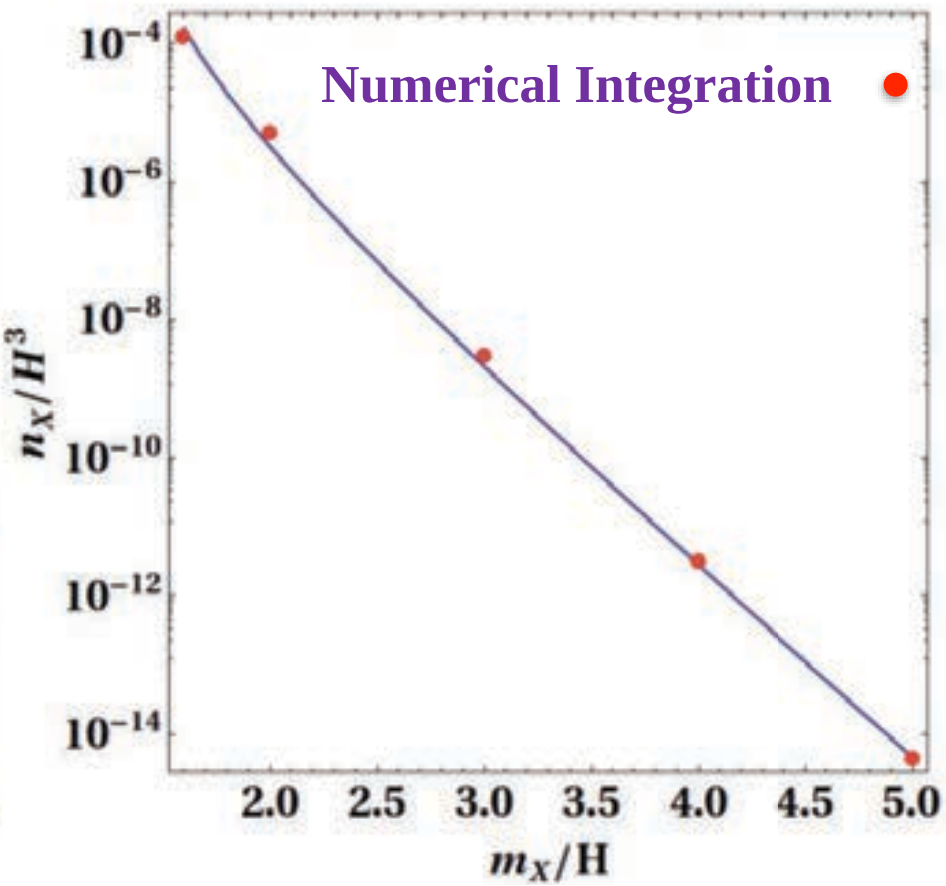
Particle Production of Inflation-Minkowski



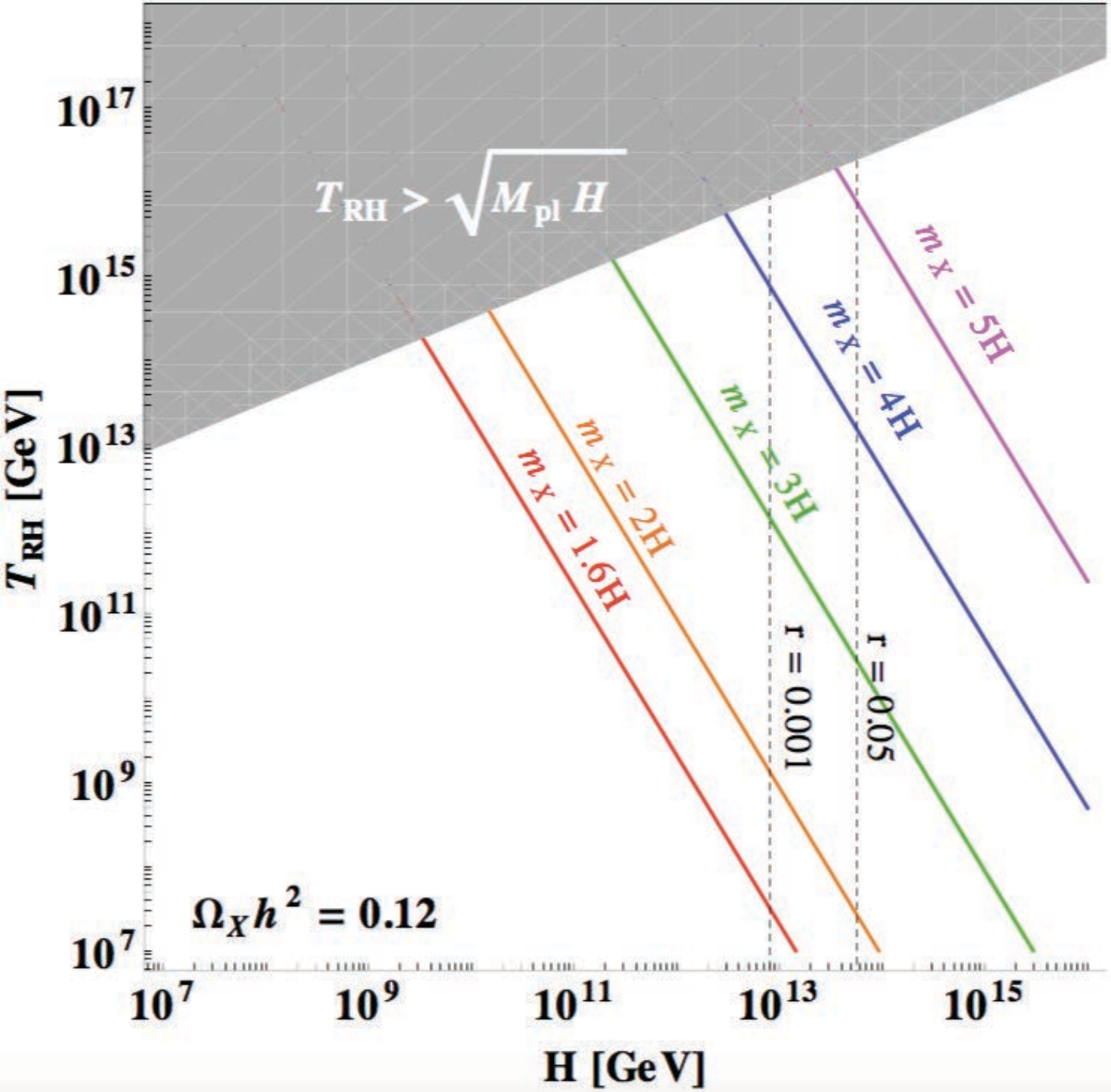


Particle Production of Inflation-Minkowski



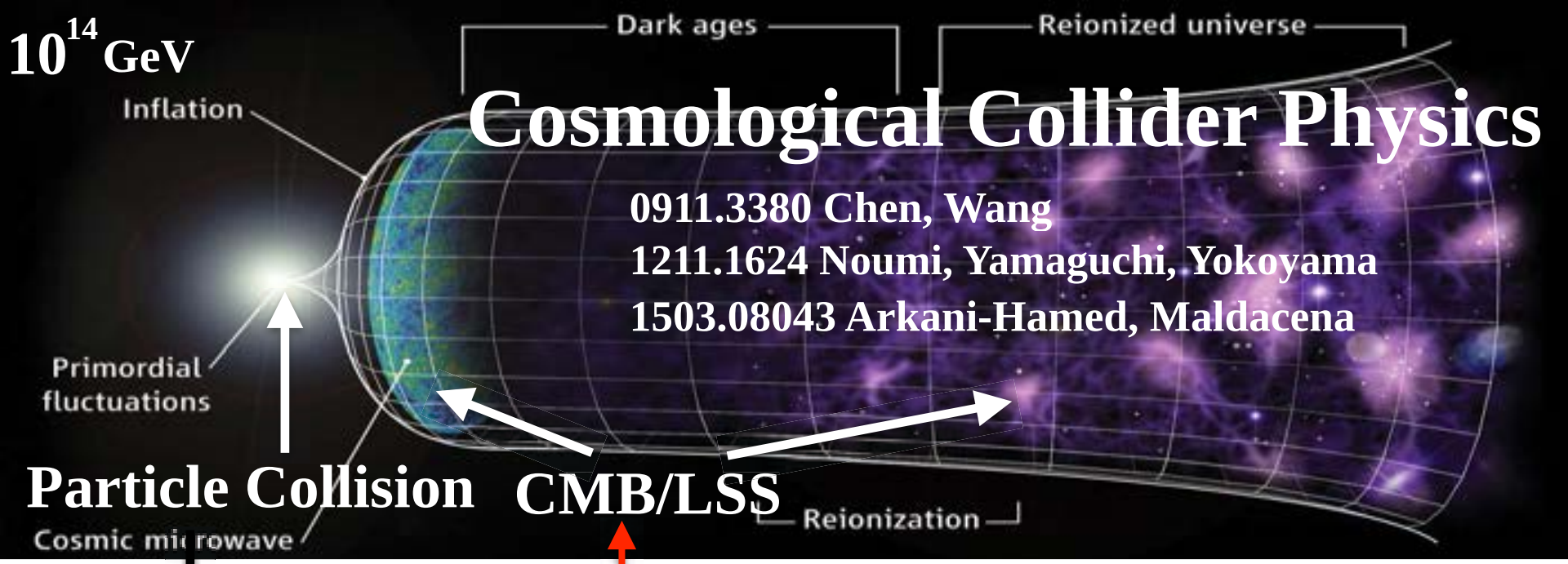


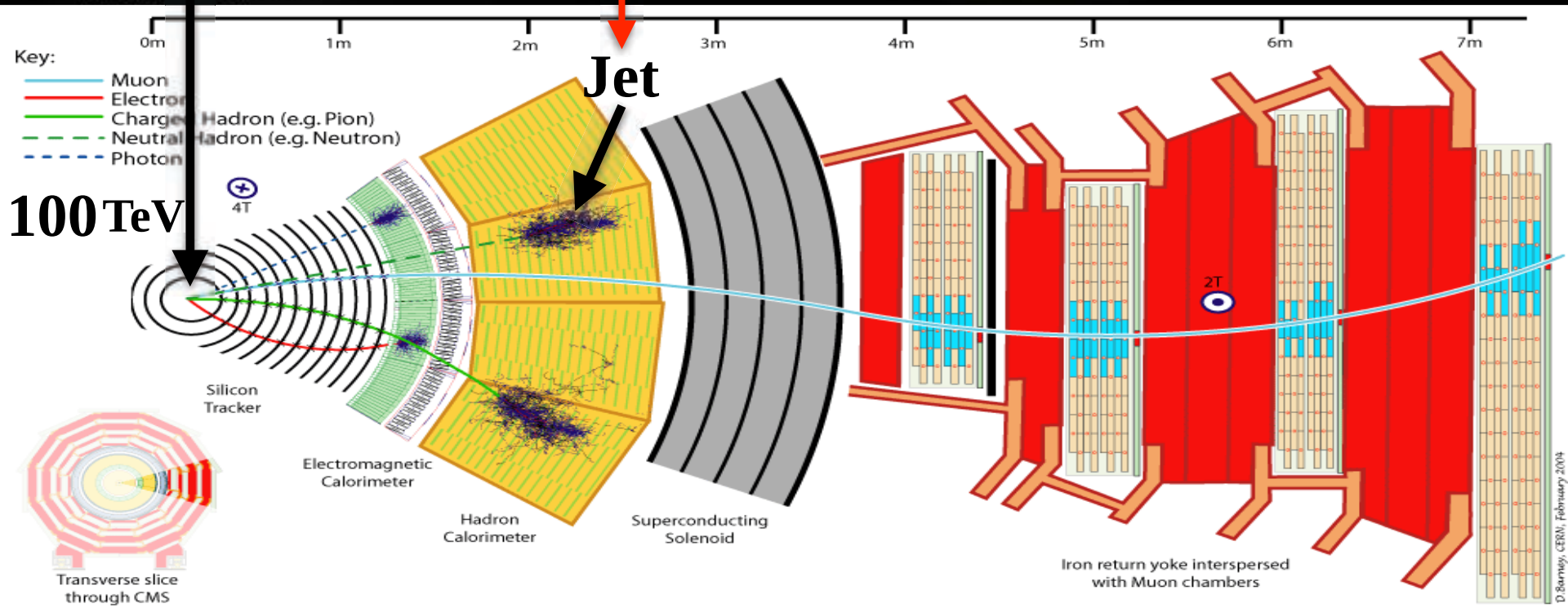
Fitted by $n_X \simeq 10^{-2} H^3 \mu e^{-2\pi\mu}$


$T_{RH} > \sqrt{M_{pl} H}$
$m_X = 5H$
$m_X = 4H$
$m_X = 3H$
$m_X = 2H$
$m_X = 1.6H$
r = 0.001
r = 0.05
$\Omega_X h^2 = 0.12$
10^{17}
10^{15}
10^{13}
10^{11}
10^{9}
10^{7}
T_{RH} [GeV]
10^{7}
10^{9}
10^{11}
10^{13}
10^{15}
H [GeV]

Cosmological Collider Physics

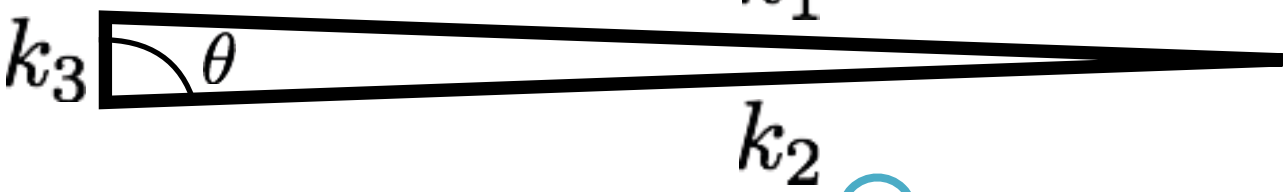
0911.3380 Chen, Wang
1211.1624 Noumi, Yamaguchi, Yokoyama
1503.08043 Arkani-Hamed, Maldacena





How do we recognise the presence of new particles from CMB or LSS?

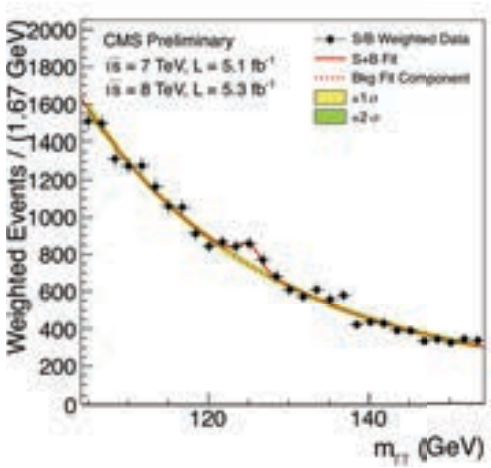
Squeezed limit non-Gaussianity distinct shape



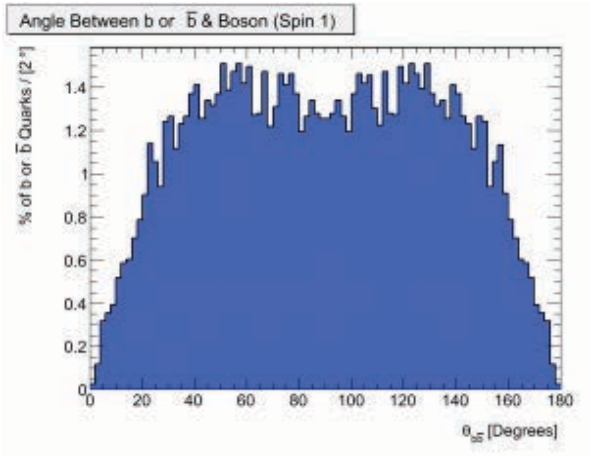
$$\frac{\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle'}{\langle \zeta_{\boldsymbol{k}_1}\zeta_{-\boldsymbol{k}_1}\rangle\langle \zeta_{\boldsymbol{k}_3}\zeta_{-\boldsymbol{k}_3}\rangle} \sim \left[F(\mu)\left(\frac{k_1}{k_3}\right)^{i\mu} + \text{c.c.}\right] P_s(\cos\theta)$$

$$F(\mu) \sim e^{-\pi\mu}, \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

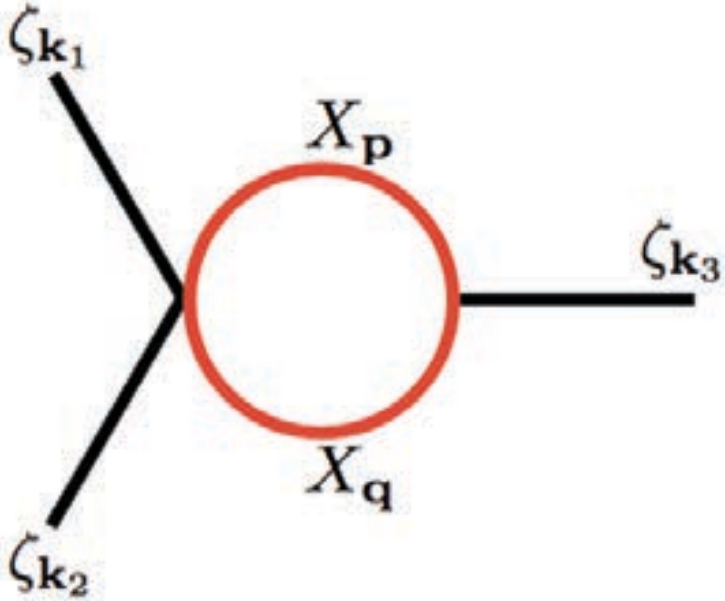
Invariant Mass

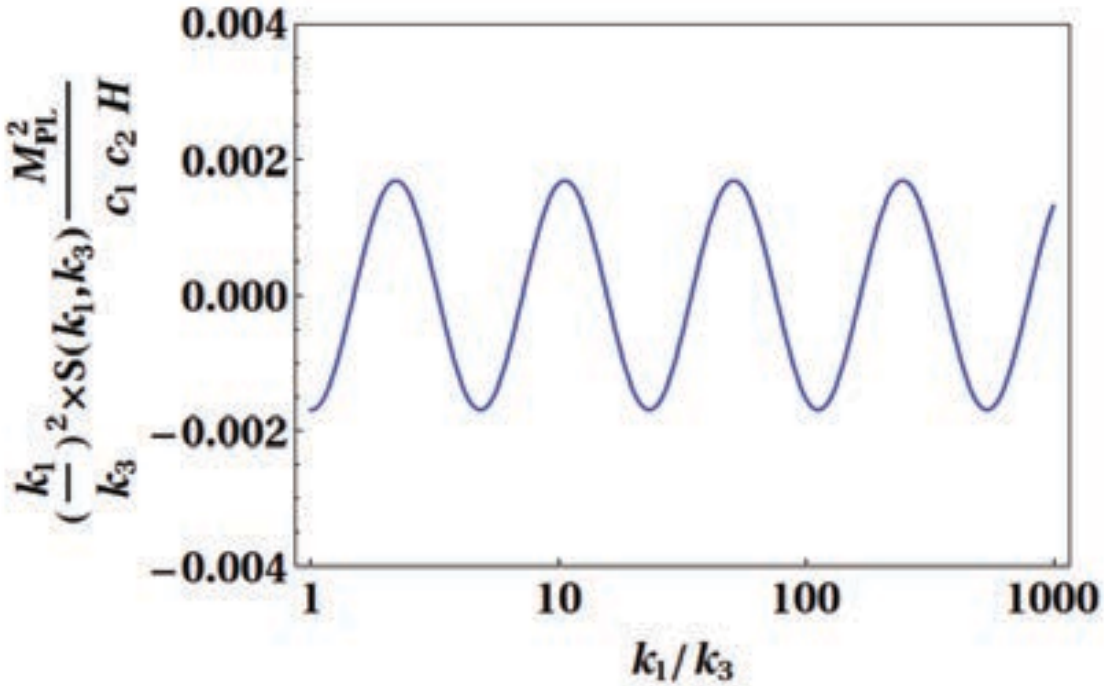


Angular Dependence



Non-Gaussianities





$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle' = c_1 c_2 \mathrm{Re}\left[g(\mu) \frac{2^{-1-8i\mu} H^5}{k_1 k_2 (k_1+k_2)^4 M_{\rm pl}^6 \epsilon^3} \left(\frac{k_1+k_2}{k_3}\right)^{2i\mu}\right]$$

$$g(\mu) = \frac{\Gamma(2-2i\mu)\Gamma(4-4i\mu)\Gamma(-2i\mu)^4}{\Gamma(1/2-i\mu)^2\Gamma(1/2+i\mu)^2} \sinh^2(\pi\mu)$$

Summary

- **Review of superheavy dark matter**
- **Review of Stokes Line Method**
- **Particle production in the toy universe**

Thank you